Enhanced Index Investing Based on Goal Programming

Includes passive management of a small number of stocks.

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Treynor and Black [1973] divide fund management into two categories: active management, to beat the market, and passive management, a buy-and-hold strategy that aims to achieve a rate of return similar to the market return, rather than to beat the market. As most active funds fail to beat the market, investors turn to passive investing to track the market return (Fabozzi and Francis [1979] and Bogle [1998]).

Index funds, which track market performance, have grown rapidly in the past three decades. Take the best known index fund, the Vanguard 500 Index Fund, as an example. It was established in 1976, and was valued at US $921 billion at the end of 2004. Its current market value is a phenomenal 8,026 times higher than when it was founded. It accounts for approximately 2.5% of the value of the total American stock fund market. In April 2004, Vanguard's market value exceeded the combined total of all the other funds in the U.S. market. Over $1,000 billion is passively invested in index funds in the United States alone (Sorensen, Miller, and Samak [1998]).

Indexing has become a favorite of the financial news media, with the rapid growth in index funds. Its most important advantage is that it gives the investor a return that approximates the market return. Note that for 1995–1998, the S&P 500 index outperformed almost 90% of all actively managed equity mutual funds.

Miller and Meckel [1999], however, argue that investors face a paradox in that:

> If most active managers typically don't add value, why pay their fees when we can match the index for a couple of basis points? Yet, if investors don't seek

incremental returns, then prices won't reflect underlying fundamentals, so it becomes easy to add value.

Enhanced index investing, which grew out of this conundrum, focuses on a risk-controlled active management strategy. Miller and Meckel describe enhanced index investing as "[tracking] the index closely, and yet some risk-controlled effort is made to add modest, reliable value relative to the index." Because enhanced index investing does not rely on aggressive active management, its cost is low compared to actively managed funds.

Two major styles of enhanced index investing are *security-level techniques* and *derivatives-based techniques* (Hill and Naviwala [1999] and Miller and Meckel [1999]). The former is a long and short strategy, where fund managers bet on long and short active positions. The second strategy is based on derivatives. An example is an option writing strategy; call options with strike prices set either at or above current market levels are sold.

Both these strategies involve active management. That is, they allocate a portion of the investment to passive management, and the remainder to active management, which poses again the question that active management underperforms in terms of market return, and that successful stock-picking is mostly due to luck and is unsustainable (Brown and Goetzmann [1995]; Elton, Gruber, and Blake [1996]; Carhart [1997]).

Can we construct a passively managed enhanced index fund without having to pay active attention to securities selection and market timing?

We address the problem by formulating enhanced index investing as a dual-objective problem. We use goal programming, which solves conflict decision problems, to construct our enhanced index portfolio.

THEORY BACKGROUND

Modern portfolio selection theory began with Markowitz [1952], who proposed the mean-variance model for constructing portfolios. He suggests that an optimal portfolio strategy can be achieved by minimizing risk for a given level of expected return, or maximizing expected return for a given level of risk. His model captures the risk aspect, and is solved by quadratic programming. Quadratic programming can deal with an optimal problem when only one objective is involved.

Instead of relying on the covariance terms needed in quadratic programming, Sharpe [1967, 1971] uses a linear objective function to approximate the quadratic objective function in constructing portfolios. Index funds, whose major objective is to minimize tracking error, can be constructed using this optimal method (Rudd [1980] and Meade and Salkin [1989]).

Different fund products, however, usually have different objectives, which are often in conflict. Grinold [1989] points out that maximizing value-added enhanced index funds is equivalent to maximizing the information ratio of the portfolio, i.e., the ratio of the active return to the tracking error. Unlike pure passive indexing, which has only one objective—to minimize tracking error—enhanced index funds have two goals: to maximize excess return (alpha), and to minimize excess risks at a controlled level. Enhanced index investing is in fact a dual-objective optimization problem, a trade-off between maximizing expected performance and minimizing tracking error.

Goal programming, first suggested by Charnes, Cooper, and Ferguson [1955], and further developed by Ijiri [1965], can handle decision problems that involve such conflicting multiple goals. Unlike linear programming, which tries to maximize or minimize the object criterion directly, goal programming seeks to minimize the unwanted deviations from any single goal. This advantage makes it useful in achieving the competing goals in constructing funds with multiple objectives.

Lee and Lerro [1973], for example, use goal programming in portfolio selection, which trades off objectives between financial risk and inflation risk. Kumar, Philippatos, and Ezzell [1978] use goal programming in portfolio selection with goals that conflict, between maximizing dividend income to their income shareholders and maximizing capital appreciation to capital shareholders to reduce their tax liability. Lai [1991] also applies goal programming in a portfolio selection problem involving multiple goals.

Enhanced index funds are designed with two objectives, namely, maximizing expected performance and minimizing tracking error. Research shows that specifying the level of active risk can affect the excess return. That is, by establishing a certain level of tracking error in order to achieve an optimal level of excess return, an optimal information ratio can also be achieved. Gupta, Prajogi, and Stubbs [1999] show that the optimal level of tracking error to maximize the information ratio is 2% to 4%, while Fabozzi, Gupta, and Markowitz [2002] say the optimal level of tracking error is 1.75% to 3.00%.

Given this theoretical background, we formulate the construction of an enhanced index fund as a dual-objective goal programming problem, where the first objective is to constrain the tracking error to be lower than 3%. The second objective is to maximize the portfolio return for optimal portfolio performance.

DATA AND METHODOLOGY

We use daily data from the Taiwan stock market, which listed 668 stocks as of the end of 2005. Stocks with incomplete data in the period were discarded, so that there were a total of 426 stocks after filtering.

The study period covers calendar years 2002-2005. We chose this period because we wanted to avoid the dramatic decline in 2000 and 2001 (see Exhibit 1). Another reason is that the market cycle in that period includes a bull market from January to April 2002, followed by a bear market until November, and fairly limited movement from then to May 2003. Bull market conditions returned, followed by another decline in market activity between March 2004 and the end of 2005.

We use a sliding windows method rather than static analysis to evaluate the dynamic performance of each metric in each test period. The advantage is that, unlike a uni-test period for a uni-training period, which makes the analysis discrete, a sliding windows method can better demonstrate the dynamic performance of the metrics in the continuous test periods.

In each sliding window, a two-year period of historical data is used for training, followed by a three-month period for performance testing (see Exhibit 2). For example, the first training period is January 2002-December 2003, and the test period is January 2004-March 2004. The second period starts three months after the first period; that is, the training period is April 1, 2002-March 31, 2004, and the test period is April 2004-June 2004.

EXHIBIT 1
Taiwan Stock Market

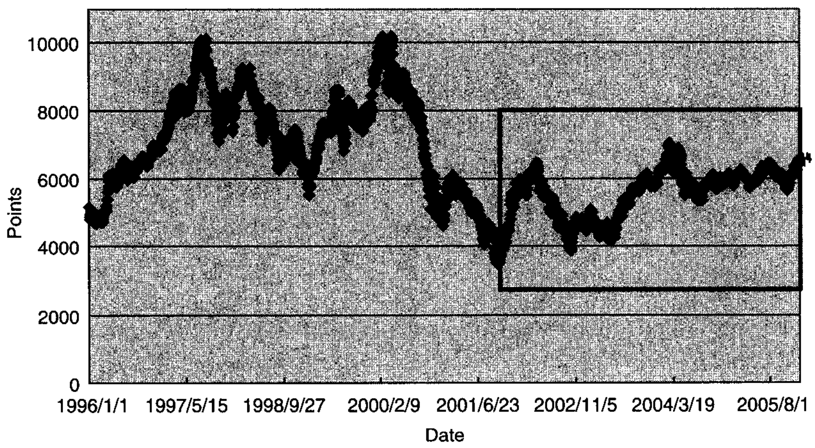
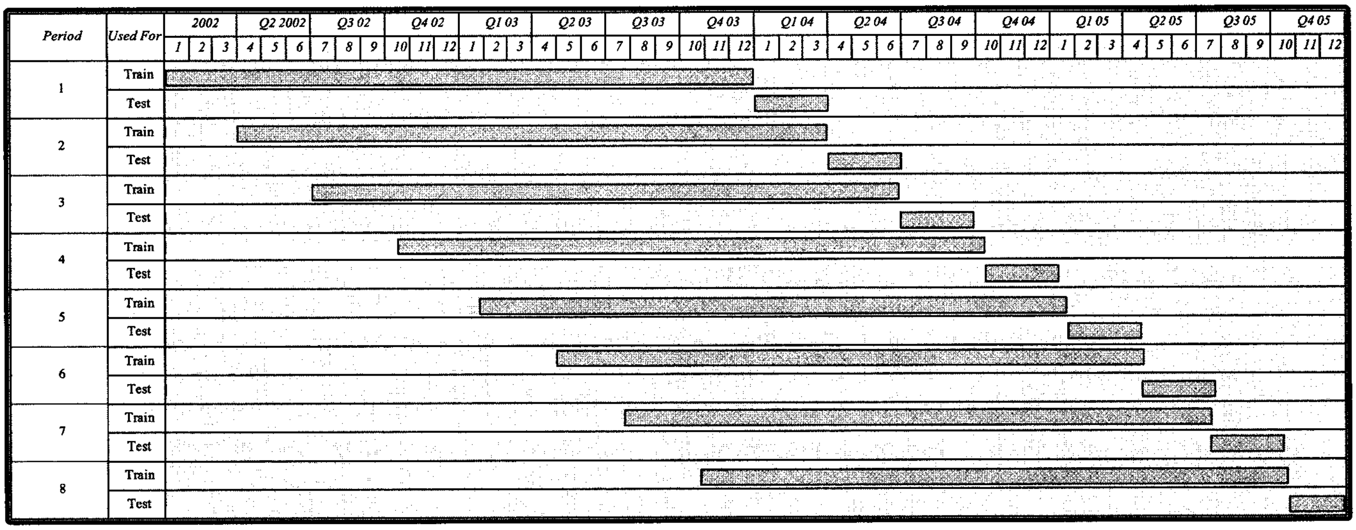


EXHIBIT 2
Data Periods



To maximize the portfolio's performance, i.e., the information ratio or the ratio of the return over the risk, we use goal programming for the dual-criterion decision problem to maximize the rate of return and to minimize the tracking error. The central idea of goal programming is to set goals, and to minimize the deviation between the goals and the outcomes. If all deviations can be reduced to zero, all the goals will be achieved. In order to minimize either under- or overachievement of a goal, a deviational variable d is assigned to the goal.

The objective function Z can be written in the form:

$$\textit{Minimize } Z = \sum_{i=1}^{k} (d_i^-, d_i^+) \tag{1}$$

$$s.t. \quad g_i(x) + d_i^- + d_i^+ = v_i \tag{2}$$

$$d_i^- \times d_i^+ = 0 \tag{3}$$

$$d_i^- \geq 0, d_i^+ \geq 0 \tag{4}$$

where (1) states that Z is a function of the unwanted deviation variables, i.e., it is the sum of the deviations of all decision goals; d_i^- represents the extent of underachievement of targets; and d_i^+ represents the extent of overachievement of targets. Thus, minimizing the unwanted deviations as much as possible means that the target value is approximately achieved. The number of goals is from 1 to k. A first-priority goal must be achieved before a second-priority goal. Z is minimized once all goals have been achieved in sequence.

Equation (2) states that, in each single goal, v_i represents the target value to be achieved for goal g_i. Equation (3) states that the deviations are either underachievement or overachievement of goal targets. Equation (4) states that both d_i^- and d_i^+ are greater than 0.

In constructing the enhanced portfolio, we have to achieve two objectives: to maximize the return, with the tracking error constraint. Following Gupta, Prajogi, and Stubbs [1999], we set the target value for the tracking error at 3%. The target value for rate of return is 7%, following the government's regulation that stocks are limited to a 7% price movement in Taiwan's stock market:

$$\sum_{i}^{n} T_i x_i + t_1^- - t_1^+ = 0.03 \text{ (target value for tracking error)} \tag{5}$$

$$\sum_{i}^{n} R_i x_i + t_2^- - t_2^+ = 0.07 \text{ (target value for rate of return)} \tag{6}$$

subject to:

$$\sum_{i}^{n} x_i = 1 \tag{7}$$

$$t_1^-, t_1^+, t_1^-, t_1^+ \geq 0 \tag{8}$$

where:

T_i: tracking error of each stock;
x_i: weight of each stock invested; a total of 1;
R_i: expected rate of return of each stock

We use three performance measures: the tracking error, the excess return, and the information ratio.

Tracking Error

The tracking error measures the deviation of the portfolio's return compared to the market return. In its standard deviation form, the tracking error is defined as the return difference between a portfolio and the market (Meade and Salkin [1990]).

The formula for tracking error is as follows:

$$TE = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (r_{fund,t} - r_{index,t})^2} \tag{9}$$

where TE is the tracking error at time t; n is the number of periods; $r_{fund,t}$ is the rate of return of the replicating portfolio at time t; and $r_{index,t}$ is the rate of return of the market at time t.

Excess Return (alpha)

Excess return is defined as the difference between the portfolio return and the market return.

Information Ratio

The information ratio is defined as the portfolio's active return (alpha) divided by the portfolio's active risk (Israelsen [2005]):

$$\textit{Information Ratio} = \frac{\textit{Active Return (alpha)}}{\textit{Tracking Error}} \tag{10}$$

EMPIRICAL RESULTS

The results for the quarterly rate of return are shown in Exhibit 3. The index fund product P is used for comparison. P is the only index fund product in Taiwan; there is no enhanced index fund in Taiwan. The average number of stocks P held in 2005 was 323.

Like P, our portfolio tracks the Taiwan weighted average stock price index. It tracks the index with a hybrid of a sampling method and an optimal method; that is, it chooses a portfolio from the entire Taiwan stock market of 668 stocks, and uses the optimal method to minimize the tracking error.

Product P also keeps a small balance for active management to enhance the investor's return. In a sense, P is a little like an enhanced index fund product. Its average market value was NTD 1.3 billion; it debuted on September 17, 2005.

In Exhibit 3, the excess return of our portfolio is positive for six of the eight periods, while the return of product P exhibits a stable negative excess return. To demonstrate how we construct the portfolio by goal programming, we plot the rate of return of the best-performing stock of the 668 stocks in each period.

Since the return of the constructed portfolio is the weighted average of all selected stocks, and there is a trade-off between the constraint on the tracking error and the maximized return, the return rate of the portfolio cannot exceed the constraint line, i.e., the top line in Exhibit 3, which represents the best-performing stocks in each period. The constraint line shows why the return of the constructed portfolio and of the market was lower than 1% in these periods; the return of the constructed portfolio moves within the constraint line and the market return line.

To determine whether the excess return of the constructed portfolio is significantly greater than zero, we use the paired t-test. The statistic shows that the p-value equals 0.017 at the 95% confidence level, which indicates that the return of the constructed portfolio is significantly greater than the market return.

Details on the number of stocks in product P and the constructed portfolio are given in Appendix A. The summary of statistics on selected stocks based on our proposed model is given in Appendix B.

In our portfolio, the tracking error varies from 0.6% to 2.9% (see Exhibit 4), because of the inherent nature of goal programming. Since our goal is both to minimize the tracking error and to maximize the portfolio's return, the number of stocks selected for our portfolio varies depending on the performance of different stocks in each period. The tracking error is influenced by the number of stocks selected, and the twofold goal.

Nonetheless, our tracking error is well below the 3% constraint. Six of the eight values show quite low tracking error—below 2%—and the lowest is actually 0.6%.

EXHIBIT 3

Quarterly Rates of Return of Best-Performing Stocks, Market, Our Portfolio, and Product P

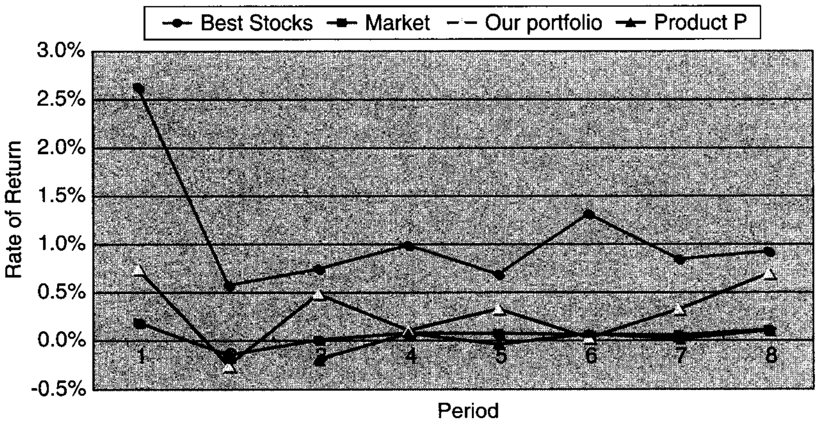
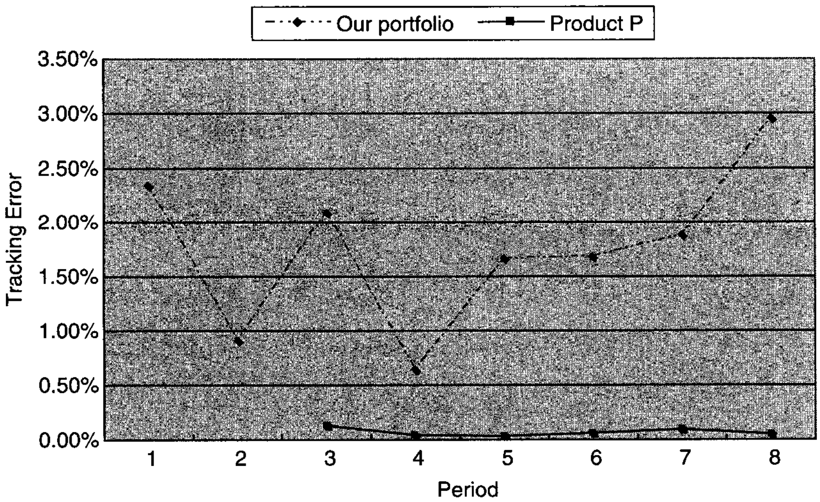


EXHIBIT 4

Comparison of Quarterly Tracking Error



The information ratio is the most important measurement for enhanced index funds. Exhibit 5 gives an analysis of this ratio on a quarterly basis. Our portfolio outperforms product P, which is unstable and shows a negative ratio on average. The drop in the IR for product P occurs because of the combined effect of a larger numerator and a rather small denominator. In Exhibit 3, product P's below-par performance occurs in periods 3 and 5, and is magnified by the denominator, especially when the denominator is between 0 and 1.

Our portfolio generates a stable information ratio, as shown by its positive information ratio for six of the eight periods, demonstrating that value is added to the fund. Lam and Lee [2005] note that "a manager is said to have outperformed if portfolio returns are higher than benchmark returns, and if the manager has delivered a positive information ratio." Kritzman and Page [2004] note that an information ratio of 0.50 is considered very high for a stock-picker.

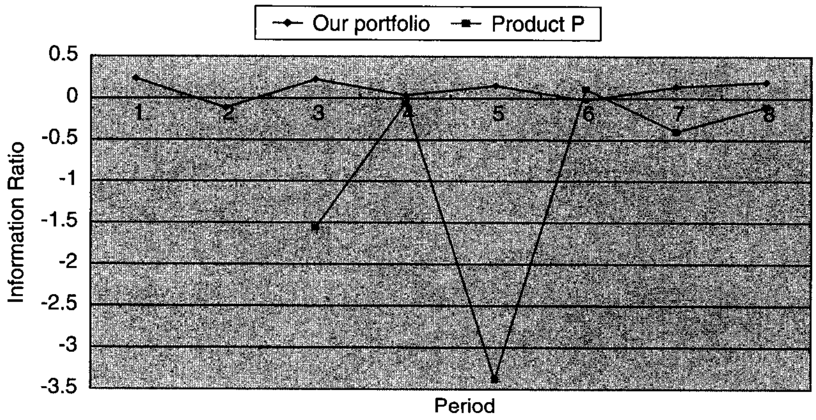
Exhibits 3, 4, and 5 show the relations among the rate of return, the tracking error, and the information ratio. Goal programming achieves the maximum return (the numerator in the information ratio measure) and the minimum tracking error (the denominator in the information ratio measure) in enhanced index funds, and balances the return and the tracking error, thereby yielding stable information ratios.

CONCLUSION

We propose a new approach to enhanced index investing that is based on a goal programming method. We formulate the construction of an enhanced index fund as a dual-goal problem. We manage the trade-off between active risk (tracking error) and active return (alpha). The results show improved performance in both the stability of excess return and the information ratio over the only enhanced indexing product in the current Taiwan financial market.

It would be interesting to examine the use of goal programming in different markets and different time periods. Sorensen et al. [2004] suggest a possible approach to exploring the risk/reward trade-off using some form of ex ante active forecasting skills. We could impose some "skill" in stock selection, such as the information coefficient (IC) or the hit rate, before weighting selected stocks to enhance the ex post return, and then compare the performance of enhanced index products. Practitioners might prefer an enhanced index portfolio with a few stocks, to obtain stable portfolio performance. Program-based procedures could be used to automate the computations.

EXHIBIT 5
Comparison of Quarterly Information Ratio



APPENDIX A
Model Performance

Period	Number of Stocks	Market Return	Portfolio Return	Tracking Error	Information Ratio
1	13	0.001886	0.007441	0.023486	0.236507
2	14	-0.00152	-0.00257	0.009095	-0.11491
3	11	8.1E-05	0.001608	0.011561	0.132133
4	15	0.0008	0.001052	0.006384	0.039479
5	10	0.000745	0.003284	0.016709	0.151989
6	10	0.000646	0.000265	0.01681	-0.02264
7	9	0.000574	0.003223	0.018778	0.141066
8	6	0.001106	0.006951	0.029605	0.197431

APPENDIX B

Summary of Statistics on Selected Stocks

Period	1		2		3		4		5		6		7		8	
Number	Code	Proportion	Code	Proportion	Code	Proportion	Code	Proportion	Code	Proportion	Code	Proportion	Code	Proportion	Code	Proportion
1	2353	*0.098*	2330	*0.093*	1102	*0.100*	2385	*0.053*	2376	*0.011*	2361	*0.010*	2361	*0.024*	2331	*0.024*
2	2331	*0.051*	2313	*0.024*	2388	*0.084*	2382	*0.133*	2369	*0.067*	2359	*0.053*	2354	*0.157*	2029	*0.005*
3	2325	*0.115*	2301	*0.077*	2850	*0.011*	2359	*0.051*	2105	*0.025*	2007	*0.045*	2337	*0.045*	1319	*0.003*
4	2324	*0.052*	1730	*0.078*	8008	*0.031*	2325	*0.071*	1101	*0.170*	9924	*0.153*	1312	*0.001*	1101	*0.005*
5	2014	*0.304*	1729	*0.007*	2325	*0.077*	2014	*0.023*	6166	*0.055*	6133	*0.000*	6142	*0.004*	2605	*0.007*
6	1905	*0.006*	1459	*0.013*	2313	*0.065*	1722	*0.041*	3007	*0.130*	3032	*0.022*	3051	*0.037*	2498	*0.956*
7	1325	*0.077*	1102	*0.061*	2303	*0.084*	1467	*0.062*	2811	*0.055*	2811	*0.022*	2887	*0.123*		
8	1101	*0.086*	5534	*0.042*	2606	*0.062*	1301	*0.113*	2610	*0.009*	2547	*0.064*	2548	*0.566*		
9	2912	*0.053*	3037	*0.021*	2542	*0.325*	1101	*0.108*	2542	*0.404*	2542	*0.560*	2498	*0.044*		
10	2606	*0.057*	2850	*0.033*	2495	*0.047*	9912	*0.033*	2437	*0.074*	2485	*0.017*				
11	2461	*0.044*	2606	*0.358*	1323	*0.114*	5525	*0.048*								
12	2439	*0.027*	2514	*0.033*			2825	*0.081*								
13	2421	*0.029*	2475	*0.053*			2618	*0.002*								
14			2411	*0.107*			2542	*0.154*								
15							2437	*0.027*								

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